A Hexagonal Box Spline Wavelet for Multiresolution Visualization of Digital Earth Data

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Abstract

Multiresolution analysis is an important tool for exploring large-scale data sets. Such analysis provides facilities to visualize data at different levels of detail while providing the advantages of efficient data compression and transmission. In this work, an approach is presented to apply multiresolution analysis to digital Earth data where each resolution describes data at a specific level of detail. Geospatial data at a fine level is taken as the input and a hierarchy of approximation and detail coefficients is built by applying a non-separable discrete wavelet transform. Multiresolution filters are designed for hexagonal cells based on the three directional linear box spline which is natively supported by modern GPUs.

Keywords: Multiresolution analysis, digital earth data, linear box-spline, hexagonal grid.

1 Introduction

Digital Earth has become an important subject in the field of Climatology. Climate research relies on large amounts of geospatial data obtained from various kinds of acquisition or simulation processes, and requires an appropriate framework to represent this data in a structured manner. In most cases, such frameworks deal with discretizing the Earth's surface into different geometric objects or cells that are used to assign data. These cells represent areas that contain geospatial information related to the point of interest. Digital Earth's geometry can consist of different types of cells. In this work we are focusing on data that are sampled on triangular cells that form a hexagonal grid (Fig-1).

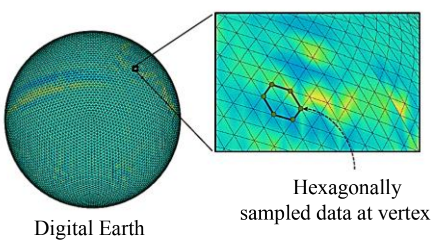

Figure 1: Data sampled on a hexagonal grid.

Owing to the vast amount of data, a level of detail (LoD) scheme is typically employed in order to facilitate data management and visualization. Such schemes allow a user to quickly browse data at a lower LoD while a higher LoD is presented on demand as the user zooms in on an area of interest. A straightforward approach to obtain different LoDs is to simply downsample the original data and store it at a number of coarse resolutions. However, this results in a drastic loss in fidelity between the levels and a lot of data duplication.

To remedy these issues, we propose to apply multiresolution analysis (MRA) to digital Earth data. Mathematically, an MRA is a nested set of linear function spaces $V^0 \subset V^1 \subset \cdots \subset V^n$, with the resolution of functions in V^j increasing with j [1]. Wavelets, a key ingredient in MRA, represent data at multiple scales and can recover data at a finer resolution from a coarser resolution via the addition of details. To further elaborate, multiresolution encapsulates two processes: decomposition and reconstruction. Decomposition is the process of downsampling some fine data F into a coarse approximation $C = [c_0,\ c_1, \dots]^T$ and some extra information $D = [d_0,\ d_1, \dots]^T$, known as the wavelet or detail coefficients. The details allow the original fine data to be perfectly reconstructed. Formally,

$$C = AF,$$
$$D = BF,$$

where A and B are wide matrices known as *decomposition filters*. Reconstruction is the process of increasing the resolution of coarse data C to produce fine data $F = [f_0,\ f_1, \dots]^T$. If there are details D available, the reconstruction process can use them to exactly reproduce the original fine data. Formally,

$$F = PC + QD,$$

where P and Q are tall matrices known as *reconstruction filters*. Together, P, Q, A and B are known as the *multiresolution filters*. Variations in these filters produce different multiresolution schemes.

This paper deals with designing multiresolution filters based on the Fast Wavelet Transform [2] that perform a MRA for hexagonally sampled digital Earth data. The next section of this paper presents the data structure used to work with digital Earth followed by a high level overview of the proposed multiresolution scheme. Finally, it concludes with some preliminary results.

2 Methodology

2.1 Atlas of Connectivity Maps

An appropriate data structure is a must when dealing with digital Earth information. In this work, we employ Atlas of Connectivity Maps (ACM) [3], which maps vertex connectivity information to separate two-dimensional arrays. Many digital Earth models are obtained by refining an icosahedron. ACM splits the icosahedron into a set of diamonds. Vertex information on the diamonds is stored in individual 2D arrays. Thus, the connectivity information of the entire digital Earth model is mapped to ten different arrays. This type of data structure provides simple and efficient ways to traverse the vertices and retrieve neighbourhood information. Fig-2 illustrates the idea of ACM applied to digital Earth.

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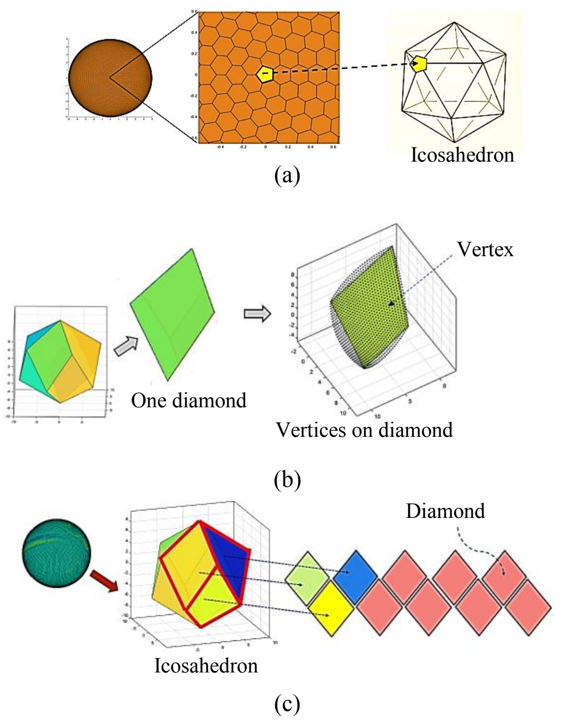

Figure 2: Atlas of Connectivity Maps on Digital Earth. (a) Generating icosahedron based on the extra ordinary vertices on the surface. (b) Retrieving connectivity information of one diamond. (c) Mapping connectivity information of all the diamonds to ten arrays.

2.2 Multiresolution Scheme

Our proposed multiresolution scheme is based on the Fast Wavelet Transform [2]. In particular, we employ a hexagonally supported three-directional linear box spline [4] as the scaling function in our MRA. Data reconstruction with this box spline is tantamount to barycentric interpolation within triangular cells which is natively supported on GPUs. In this multiresolution scheme, the decomposition and reconstruction steps are efficiently performed via discrete convolution operations. The decomposition step is performed as follows:

$$C = (((F * u) * a)_{\downarrow 2}) * a^{-1},$$
$$D = (F * \mathring{w})_{\downarrow 2},$$

where F is the fine resolution, and C and D are coefficients and details respectively. Moreover, u is a scale filter obtained by leveraging the self-similarity of the linear box spline [5], and the auto-correlation filter a and its inverse a^{-1} perform an orthogonal projection of the fine level down to the coarse level. The weight filter $\mathring{w}$ is obtained by exploiting the fact that the detail space is the orthogonal complement of the coefficient space with respect to the fine level.

The reconstruction step follows a similar method:

$$F = (C_{\uparrow 2}) * u + (D_{\uparrow 2}) * w,$$

where F is the fine level which is reconstructed perfectly from coarse level coefficients C and details D. The filter w is obtained from $\mathring{w}$ by exploiting the fact that w and $\mathring{w}$ form a biorthogonal wavelet.

This multiresolution scheme is applied separately to each diamond. Coherence between the diamonds can be achieved by imposing appropriate boundary conditions for the convolution operation.

3 Preliminary Results

We have tested our proposed scheme on ICON (ICOsahedral Non-hydrostatic) datasets. The ICON format is a joint venture of the German Weather Service (DWD) and the Max-Planck-Institute for Meteorology (MPI-M), and is used for numerical climate simulations. The ACM data structure is applied to this dataset and the connectivity information of ICON is stored in 2D arrays. Our multiresolution scheme is also applied to ICON to observe the results of the decomposition and reconstruction steps. Figure 3 shows some results of our implementation.

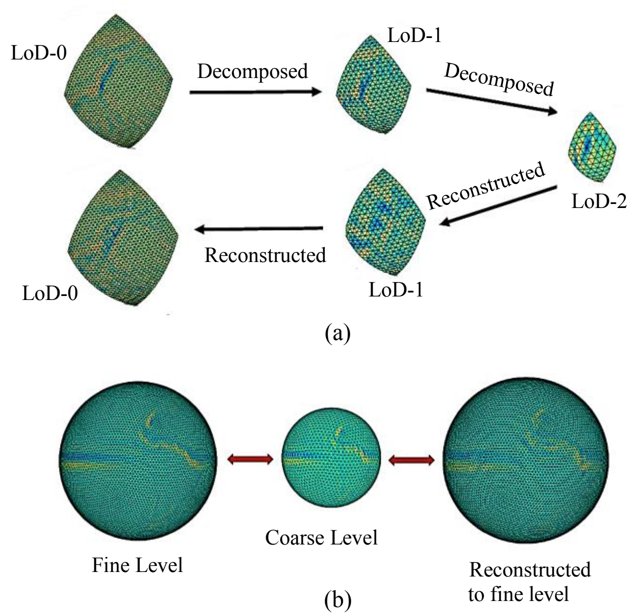

Figure 3: Results of applying multiresolution scheme on ICON Digital Earth data. (a) Shows decomposition and reconstruction of different level on a single diamond of ICON, whereas (b) shows the multiresolution analysis of the entire digital Earth model.

4 Conclusion

This paper presents a work in progress where a wavelet is developed for multiresolution visualization of digital Earth. Future work includes investigating the error due to the decomposition step, and the impact of different boundary conditions. We believe that successful completion of this wok will provide an efficient multiresolution scheme that can be applied to hexagonally sampled data for LoD visualization as well as data compression.

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